Math Assignment Unit 5

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Task 1: Understanding Exponential and Logarithmic Functions

(i) Definitions, Relationship, Key Factors, Domain, and Range

Exponential functions are mathematical functions of the form $f(x)=a\cdot b^x$, where a is a constant (often representing the initial value), $b>0$, and $b/1$ is the base (Abramson, 2023). The variable x is the exponent, which makes these functions grow or decay at a rate proportional to their current value. For example, $f(x)=2^x$ is an exponential growth function because its base (2) is greater than 1.

Logarithmic functions are the inverses of exponential functions. They are defined as $f(x)=\log_b(x)$, where $b>0$, $b/1$, and $x>0$ (Abramson, 2023). This means that if $y=b^x$, then $x=\log_b(y)$. In other words, logarithms answer the question: “To what power must we raise the base b to get x ?”

The key variables are:

- In exponential functions: a (initial value), b (base), and x (exponent).
- In logarithmic functions: b (base), x (input, must be positive), and the output is the exponent.

Domain and Range:

- For exponential functions $f(x)=a\cdot b^x$:
  Domain: All real numbers (R)
  Range: $(0,\infty)$ if $a>0$; $(-\infty,0)$ if $a<0$
- For logarithmic functions $f(x)=\log_b(x)$:
  Domain: $(0,\infty)$
  Range: All real numbers (R)

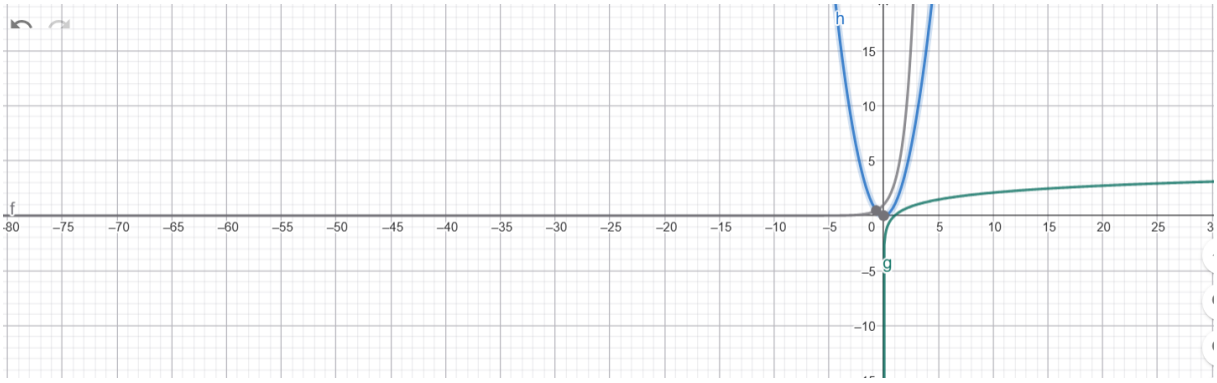
These functions are inverses, so their graphs are reflections of each other across the line y=x .

(ii) Differences Between Exponential, Logarithmic, and Power Functions

Function Type	**General form**	**Example**	**Growth Pattern**	**Special Points**
Exponential	f(x)=a·bx	f(x)=3x	Rapid growth (ifb>1)	Horizontal asymptote:y=0
Logarithmic	f(x)=logb(x)	f(x)=log3(x)	Slow growth, increases without bound	Vertical asymptote:x=0
Power	f(x)=x	f(x)=x2	Polynomial growth	No asymptotes; vertex at

	n		(depends on exponent)	origin for even n

Graphical Illustration:



(iii) How to Explain if a Function Has Exponential Growth

A function exhibits exponential growth if its rate of change is proportional to its current value — meaning it grows faster as it gets larger. Mathematically, this is modeled by a function of the form $f(x)=a\cdot b^x$ with $b>1$ (Abramson, 2023).

In practical terms, if doubling the input doesn't double the output — but instead multiplies it by a fixed factor (e.g., population doubles every year), then it's likely exponential.

For example, if a population grows by 5% annually, next year's population = current × 1.05 — this multiplicative pattern defines exponential growth.

(iv) Which Grows Faster: Exponential or Logarithmic?

Exponential functions grow much faster than logarithmic functions. While logarithmic functions increase very slowly (even as inputs become large), exponential functions accelerate rapidly. For instance, compare $f(x)=2^x$ and $g(x)=\log_2(x)$:

- At $x = 8 : f(8) = 256 , g(8) = 3$
- At $x = 16 : f(16) = 65,536 , g(16) = 4$

This shows that exponential growth outpaces logarithmic growth dramatically as x increases. In fact, for any base $b>1$, bx eventually surpasses $\log b(x)$ no matter how large x gets (Abramson, 2023).

(v) Observations from Graphs

Using GeoGebra, I graphed the examples from part (ii): $y=3^x$, $y=\log_3(x)$, and $y=x^2$.

Observations:

- Exponential ($y=3^x$):

    - Passes through (0,1) — y-intercept.
    - No x-intercept.
    - Horizontal asymptote at $y=0$.
    - Steep upward curve for $x>0$.

- Logarithmic ($y=\log_3(x)$):
    - Passes through (1,0) — x-intercept.
    - No y-intercept (undefined at x=0).
    - Vertical asymptote at $x=0$.
    - Very slow rise for $x>1$.
- Power ($y=x^2$):
    - Parabola opening upward.
    - Vertex at (0,0).
    - Symmetric about y-axis.
    - No asymptotes.
    - These visual patterns reinforce the theoretical differences discussed above.

Task 2: Applying Logarithmic Properties

(i) Prove that $\log_6\left(\frac{216^x}{1296^x}\right) = -x$

We start by simplifying the expression inside the logarithm.

First, note that:

- $216 = 6^3$
- $1296 = 6^4$

So,

$$\frac{216^x}{1296^x} = \frac{(6^3)^x}{(6^4)^x} \downarrow \frac{6^{3x}}{6^{4x}} = 6^{3x-4x} = 6^{-x}$$

Now apply the logarithm:

$$\log_6\left(6^{-x}\right) = -x \cdot \log_6(6) = -x \cdot 1 = -x$$

Therefore,

$\log_6\left(\frac{216^x}{1296^x}\right) = -x$ is proven.

(Used properties: Power Rule $\log_b(a^c) = c \cdot \log_b(a)$, and $\log_b(b) = 1$.)

(ii) Prove that:

$$\log(x+1)^2 + \log(2x-1)^3 - \log(x)^2 - \log(2x-1)^4 + 6\log(x+1) = \log\left(\frac{(x+1)^8}{x^2(2x-1)}\right)$$

We combine terms using logarithmic properties:

Recall:

- $\log A + \log B = \log(AB)$
- $\log A - \log B = \log(A/B)$
- $n \log A = \log(A^n)$

Group like terms:

Left-hand side:

$$[\log(x+1)^2 + 6\log(x+1)] + [\log(2x-1)^3 - \log(2x-1)^4] - \log(x)^2$$

Apply power rule to combine:

$$\log((x+1)^2) + \log((x+1)^6) = \log((x+1)^{2+6}) = \log((x+1)^8)$$

$$\log((2x-1)^3) - \log((2x-1)^4) = \log\left(\frac{(2x-1)^3}{(2x-1)^4}\right) = \log\left(\frac{1}{2x-1}\right)$$

Now combine everything:

$$\log((x+1)^8) + \log\left(\frac{1}{2x-1}\right) - \log(x^2) = \log\left(\frac{(x+1)^8}{2x-1}\right) - \log(x^2) = \log\left(\frac{(x+1)^8}{x^2(2x-1)}\right)$$

Thus, the identity is proven.

(iii) Solve for x : $10e^{2x-3} = 15e^{5x-7}$

Step 1: Divide both sides by 10:

$$e^{2x-3} = \frac{3}{2}e^{5x-7}$$

Step 2: Divide both sides by e^{5x-7} :

$$\frac{e^{2x-3}}{e^{5x-7}} = \frac{3}{2} \Rightarrow e^{(2x-3)-(5x-7)} = \frac{3}{2} \Rightarrow e^{-3x+4} = \frac{3}{2}$$

Step 3: Take natural logarithm of both sides:

$$\ln(e^{-3x+4}) = \ln\left(\frac{3}{2}\right) \Rightarrow -3x + 4 = \ln\left(\frac{3}{2}\right)$$

Step 4: Solve for *x* :

$$-3x = \ln\left(\frac{3}{2}\right) - 4 \Rightarrow x = \frac{4 - \ln\left(\frac{3}{2}\right)}{3}$$

Numerically, ln(1.5)≈0.4055 , so:

$$x \approx \frac{4 - 0.4055}{3} \approx \frac{3.5945}{3} \approx 1.198$$

Final Answer: $x = \frac{4-\ln(3/2)}{3}$ or approximately 1.198

Task 3: Modeling Cancer Cell Growth

(i) Table of Yearly Increase (2018–2023)

Initial cells in 2018: 232.26
Growth rate: 2% per year → multiplier = 1.02

YEAR	NUMBER OF CELLS (ROUNDED TO 2 DECIMALS)
2018	232.26
2019	232.26 × 1.02 = 236.91
2020	236.91 × 1.02 ≈ 241.65
2021	241.65 × 1.02 ≈ 246.48
2022	246.48 × 1.02 ≈ 251.41
2023	251.41 × 1.02 ≈ 256.44

(ii) Mathematical Function Representing Growth

This is exponential growth since the quantity increases by a fixed percentage each year.

Function: $C(t)=232.26\cdot(1.02)^t$
Where:

- $C(t)$ = number of cancer cells after t years since 2018
- Base = 1.02 (growth factor)
- Initial value = 232.26
- Time t is measured in years from 2018

This matches the general form $f(t)=a\cdot b^t$ (Abramson, 2023).

(iii) Project Level in 10 Years

Set $t=10$:

$$C(10) = 232.26 \cdot (1.02)^{10}$$

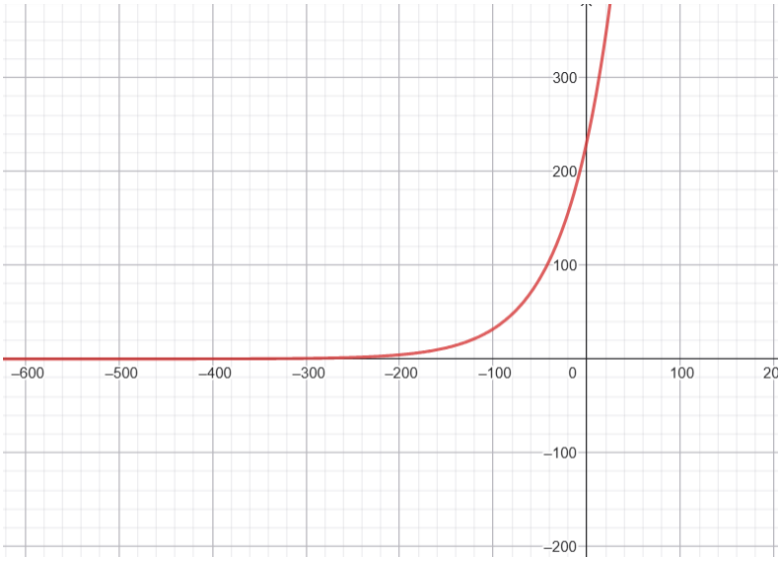
Calculate $(1.02)^{10} \approx 1.21899$

$$C(10) \approx 232.26 \times 1.21899 \approx 283.14$$

After 10 years, there would be approximately 283.14 units of cancer cells.

(iv) Graph of Growth Pattern

To illustrate this, I plotted the function $C(t)=232.26\cdot(1.02)^t$ using GeoGebra.



References

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